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BY

FLORENCE MILNER

DETROIT UNIVERSITY SCHOOL

DETROIT, MICHIGAN

BOSTON, U.S.A.

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1900

ON TEACHING GEOMETRY.

Of writing many geometries there is no end. With any of them, or without them all, the good teacher will get good results; with the best of them, the poor teacher cannot rise above mediocrity. Under both conditions, however, there is wisdom in a careful choice, for a strong book not only lessens the labors of a good teacher, but makes it possible for a class to get some value out of the work in spite of poor teaching. Yet we as teachers are inclined to ask too much of text-books, and we expect them not only to do their own work, but also to become responsible for a large share of ours. It is the province of the text-book to present clearly, according to its established sequence, the subject matter of geometry, not to teach how to teach it, and that book is best which has in it least of anybody's method, even of the present writer's, and most of clearly expressed geometry. So, however good the book, there always remains, and wisely too, much for the teacher to do.

It is not the present purpose to outline any particular method of teaching, but to call attention to a few general lines which, in harmony with the

subject, should be followed out, and to touch upon some points in a wider training which a correct handling of the subject should give.

Geometry differs markedly from the preceding studies, and success or failure depends largely upon getting at the beginning the right point of view. To play a game of chess one must learn the moves of individual pieces. These, in infinite combination, but always under the same fixed rules, make up the most intricate of intellectual games. Geometry is analogous. We expect no one to play a game of chess until he learns the moves; we should not expect a pupil to work intelligently in geometry until he is helped to the mental attitude demanded by the subject, and knows something of the few simple truths that are the guiding thread through the seemingly intricate labyrinth.

In geometry the pupil encounters for the first time formal logic, and we are too prone to plunge a class into it without adequate preparation. Before opening the book at all, the pupil should be taught something of the nature of logic, something of its requisites, something of its method of work. Certain texts adhere to set, formal demonstrations, others give nothing but original work, while between these two extremes are all grades of compromise, with more or less practical application of established truths. Whether we are working out original theorems or are following the demonstra-

tion of a conventional proposition or are using our knowledge in the solution of a practical problem, the process is identical.

Axioms and definitions are the foundation upon which the whole superstructure of geometry is builded, and in the beginning a class should be thoroughly grounded in an understanding of their nature and province. Definitions should receive first attention.

If you and I are to carry on a discussion about an apple, we must agree as to its characteristics. If you define it as "the fleshy pome or fruit of a rosaceous tree," while I insist that it is a hard, black mineral, we shall never get far in our argument. If you say that "a triangle is a plane figure bounded by three right lines," while I call a solid bounded by four curved surfaces a triangle, our discussion will again come to grief. It becomes necessary, then, that you and I agree upon a common conception of the object under consideration, and that we shall so describe it that ambiguity is impossible. This is the province of the definition. A definition is such a description as will bring up to all minds the same conception. The definitions in geometry have long been agreed upon by mathematicians, and the importance of knowing, and knowing with verbal accuracy, the universally accepted description of geometric concepts cannot be too strenuously insisted upon, and the teacher who in place of accurate technical lan-

guage accepts careless and verbose statements, does a class a great wrong. A lover of literature will have little patience with one who, pretending to give Hamlet's soliloquy or Milton's sonnet *On his Blindness* would dare substitute his own meagre words for the matchless language of the great masters. In certain directions the phraseology of mathematics has crystallized equally with other literature, and the giving of familiar definitions is not the place for original work,—that comes later.

The attitude toward axioms should next be made right. Every sane mind finds itself unconsciously in possession of certain knowledge. We know that certain things are true. No one can remember when he learned that the whole of anything is greater than any one of its parts, but the babe creeping along the floor has a practical knowledge of the truth, and he will contest it with you to the limit of his physical strength. Later he will formulate the idea, and possibly when he gets into the high school, some teacher will set him to learning this axiom as though it were something new. On the contrary, the youth should be taught that certain facts are so evidently true that everybody must needs accept them or be counted of unsound mind. When he begins the study of geometry, many of these truths should be put into compact form for future use. Here again comes the value of systematic and even stereotyped phraseology. He should learn now to state in the technical lan-

guage of mathematics what he and everybody else have long known.

When the pupil has been put in the right attitude toward axioms and definitions, he is ready to open his geometry and learn how to use them. Certain general definitions should be studied. He should know what a proposition is; that there are various kinds, differing in purpose and in form of expression. As he comes to them, he should discriminate carefully between theorem, problem, corollary, and lemma.

Most geometries give some preliminary work consisting largely of definitions and other discussion of geometrical concepts. This should be studied with more or less thoroughness, as the preparation of the class demands.

Entering upon the work peculiar to geometry, we come to the opening section, which may treat of perpendicular straight lines, of triangles, or of whatever the sequence of the particular text in use demands. After reading the first theorem, the hypothesis should be isolated that the pupil may know exactly what is to be his by gift of this same hypothesis, and he should be taught to enumerate these gifts. Perhaps the enumeration will run like this: "A line, a point without that line, and a perpendicular from the given point to the given line." Show the class that it is not sufficient to say that a line and a point are given; this allows one to locate the point within the line. Whenever they

fail to be thus accurate in statement, draw figures, taking such latitude as they leave you by inaccurate statement. A few illustrations will show them the necessity of talking sharply to the facts, and the pupils will soon learn to hold each other to statements that admit of no ambiguity.

A certain power of reconstructive imagination should also be cultivated. Besides stating accurately the conception that is in one's own mind, there must be the ability to construct, rapidly and clearly, mental pictures of all statements made in class. Every sentence uttered should add something to the picture or present some new phase of it. Every recitation should be a series of constantly changing views in which everybody, in his mind's eye, sees the same things. This result is not easy to obtain, but it is possible, and the degree of excellence reached by a class becomes a crucial test of the teacher.

Impress upon a class that the formal statement of a proposition is always a *general* truth, true of all figures falling under the given conditions. These statements are conventional and should be as accurately stated as definitions and axioms.

In proving a proposition, our human limitations require us to fix our minds upon a particular case, and accordingly a *special* statement follows.

Given 1. Line AB.

2. Point P without the line.

3. Perpendicular PD upon the line

To prove, that *PD* is the shortest distance from *P* to the line *AB*.

If there is more than one conclusion, bring out the facts in clear, mathematical language. Do not be satisfied to let a class say, "To prove that from the point *P* one perpendicular and only one can be let fall to the line *AB*." It should be stated, "To prove: first, that one perpendicular can be let fall upon the line *AB*; and second, that only one can be let fall from the same point upon the line."

This may seem to some like splitting hairs, but mathematics is an exact science, and in the beginning too much cannot be done in training toward accurate thought and exact expression.

Now comes important work in preparing for a formal demonstration. Here the class must learn that the regular, logical form of every argument is a syllogism. Pupils will not be frightened at the new word, and they will like to use it when they comprehend its meaning. Discuss it with them. Explain the major premise, the minor premise, and show how the conclusion inevitably follows. Give them an example and set them to hunting for others. You will wonder where they find so many.

Next teach them that every step in a well-constructed demonstration is taken by means of a syllogism. As the major premise must always enunciate a general truth, the major premise in their first demonstration must be either an axiom or a definition, for these are the only general geometric

truths in their possession. The minor premise isolates and states formally the special case now present and in harmony with the major premise. The conclusion is the new knowledge revealed by bringing these two premises together. This conclusion in turn may become the minor premise, and so on to the end of the argument.

Show a class how a syllogism may be worked out from some of the early definitions and axioms, letting the minor premise be a fact given. Tell them, for instance, to draw the line *CD* to the point *D* in the line *AB*, making the angle *CDB* a right angle. This is the fact given, and if demonstration were to follow, might constitute the hypothesis. Now if the fact is of any value in constructing a syllogism it must fall under some general definition or axiom. Some one will discover that the definition for a perpendicular fits the case. Then you can show how the inevitable conclusion that *CD* is perpendicular to *AB* must follow. Finally show the formal structure of a syllogism and write out the one developed.

1. The line *CD* meets the line *AB*, making the angle *CDB* a right angle.

2. A perpendicular to a line is a line which makes right angles with a given line.

3. *CD* is perpendicular to *AB*.

Repeat the process until you make them see that when the first two terms are rightly selected and agreed upon, there is no escape from the

conclusion. It follows like a decree of inexorable fate.

Every demonstration is a chain of syllogisms, in all of which the major premise must be some general conclusion already established, either axiom, corollary, theorem, definition, or algebraic truth. Of course as the work goes on, the formal statement that involves frequent repetition can be contracted, but the habit of looking at every demonstration from this side is invaluable.

Text-books do not follow this form, for they rarely give complete demonstrations. The text is an outline of the subject, a note-book, keeping the general trend of the argument but leaving the pupil to fill in the suggested discussion. Teachers rarely insist upon so closely logical a demonstration as is here outlined, but experience and careful comparison have convinced me that the syllogistic method closely adhered to in the beginning produces results that come from no other kind of work.

In the illustration just used, a pupil might say and with truth, "*CD* is perpendicular to *AB* because *CDB* is a right angle," and "When a line meets," etc., but that is making a statement and then going back to try to make the statement good. The other method never leaves a chance to question a point, for one follows another in the logical order that carries conviction at every step.

If you announce to an opponent the thing of

which you expect to convince him, the effect is usually to arouse his antagonism, and he will then and there make up his mind, and no argument of yours, no matter how convincing, is likely to change his opinion. He may listen to you politely, but when you are through he will probably say, "Yes, but as I said in the first place." Don't give him a chance to say anything in the first place, but make him grant your premises, one after another, then when your conclusion is reached, there is nothing left for him to do but accept it.

If this method is used in conventional and formal demonstration, it becomes of greatest value in original work. A pupil thus trained will, when given original work, have a definite and efficient method of attack. He will first study his hypothesis, isolating each individual part of it. Taking one fact for a minor premise, he will next examine his stock in trade and see what definition, theorem, corollary, or other authority he can bring to bear on the case in hand. He will draw his conclusion and see if it advances his argument or brings him nearer the desired end. If it does not, then he decides that his minor premise is wrong and returns to his hypothesis for another fact, and constructs the syllogism. Sooner or later he is bound to reach the right conclusion.

Sometimes teachers allow pupils to say that such or such a thing is true "by a previous proposition." This is a much abused and greatly over-

worked expression. It becomes to a poor student like a cloak of charity, and if permitted, will be used to cover a multitude of hazy, illogical, and vague ideas.

Do you think that a judge in court would admit as evidence the statement that somewhere, sometime, somebody had made a decision in a case somewhat similar to the one in hand? Indeed not. The lawyer would have to produce his authority, giving title, volume, and page of report, with date of decision. In all demonstrations equal accuracy should be demanded. Not that chapter and verse be given, but pupils should be made to quote geometric scripture in proof of every conclusion. And "quote" here means exactly what the word indicates, not a slipshod attempt at giving the idea.

There are certain definite and wide-reaching results that should come from the right teaching of geometry. Mention has already been made of the importance of accurate and exact expression, but this must be preceded by equally exact and accurate thinking. Better than any other subject, geometry will train the youth to keep close watch and ward over the action of his mind and accustom him to express clearly and honestly the result of his own mental happenings. The ability to make wise selection under varying circumstances, is repeatedly demanded. After a figure is drawn, the pupil should examine it carefully. He may discover in it many things which he knows, from

construction, hypothesis, or other conditions, are all true, but only one of them has any concern with the business in hand. He should be trained to see that one fact, and be able to make proper use of it. Teach him to go straight after the one that he needs, and make it serve him.

He should also be taught that certain things lie within his power to do, while others are as absolutely beyond his control as are the turbulent waves, the floating clouds, or the sweeping course of the planets.

He may push a stone from the edge of an overhanging cliff, but after that he must let it go crashing down the mountain side. He may elect to let fall a perpendicular from a certain point to a given line, but he cannot dictate where it shall strike the line. He may know certain things about two figures, and it may be wise to try the application of one to the other. He may lift one figure and place a line of it or an angle upon its equal, but there his control over the matter ceases. From that moment he is under law, and face to face with *the eternal.* Let him stand there a humble spectator, knowing that till heaven and earth shall pass, "one jot nor one tittle shall in nowise pass from the law" over which his finite mind has no control. He was responsible for placing the figures together, but before the resulting consequences he is helpless. He may watch to see if in the finality there is anything that concerns him. Here again he will

discover various things accomplished; but again he must recognize the one conclusion for which he has been striving, must isolate it from the rest, and hold it up to view with the strength of conviction.

The young mind is naturally unreasoning, and often utters words without consciousness of a definite idea back of them. The pupil will watch the teacher rather than himself, to determine whether he is travelling the right road. It is very easy for a teacher unconsciously to take this responsibility, and by various gentle leading-strings keep the pupil in the path; but such work is not teaching geometry. If a class leans upon you, or has the habit of watching you, rest assured that your teaching is not right. If the development is correctly carried on, the pupil will be aroused to watch his own mind, his own statements, and the work in hand; the more nearly he can forget *you*, the better. He must learn to discover what is actually happening in his own mind as the process of reasoning goes on; he must be trained to faith in his own convictions, and to fearlessness in expressing them. Nothing is more pitiful than a mind that can be shifted from any position by an incredulous question. At first your pupils will not think independently. Rouse them from this condition, and force them to a conclusion of some sort. A wrong opinion is better than no opinion at all, for activity is better than stagnation.

In a demonstration the teacher must be con-

stantly in the attitude of the doubting Thomas, who took nothing on faith, but always demanded evidence. In this way the burden of proof rests with the pupil, and his aim should be to anticipate every possible question. An exceptionally good teacher of geometry says that the right results have not been attained until a pupil not only ceases to lean upon a teacher, but is also able to stand up against a teacher. Do not rest until you see your pupils thinking independently, and at the same time clearly, fearlessly talking out their conclusions.

For a complete setting in order of one's mental house, the High School course offers nothing better than geometry. The facts learned have some value, but the greatest good is the mental poise, the clearness of vision, and the honesty of expression that it develops. In addition to all this, a pupil rightly trained will learn to measure his own strength, will recognize his limitations, and will bow in reverent respect before some things greater than himself.

Note. — Some teachers may be interested to see how the preceding discussion would tend to elaborate in recitation the demonstrations as they appear in our best textbooks. It has seemed wise, accordingly, to add such a detailed demonstration. The particular theorem is selected because it offers illustrations of more points than does any other in the early part of the work.

The syllogistic method has been strictly adhered to until the last few steps of the concluding argument are reached. At this point the mind works so rapidly as to become impatient of talking out in slow words what it grasps instantly, if the preceding argument has been properly built up. You have seen children stand dominoes in a row at regular intervals, working with painstaking care to get them rightly placed. A single touch upon the last one overthrows it, and the others inevitably fall in rapid succession. So in this argument. When the conclusion that CEC cannot be a straight line is reached, all the rest of the structure comes tumbling down so rapidly that no time is left to do more than to watch each domino as it falls. See to it that the arguments are rightly placed as the demonstration is built up, and the rest will take care of itself.

Of course the syllogisms can be easily repeated in this last part as well, and it is often a good exercise to allow a class to supply them.

BOOK I. PROPOSITION VI. THEOREM.*

From a given point without a straight line a perpendicular can be drawn to the line, and but one.

Given the line AB and the point C without the line.

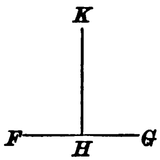
To prove (1) that from the point C one $\perp$ can be drawn to the line AB;

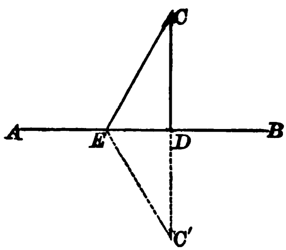
(2) that only one $\perp$ can be drawn from the point C to the line AB.

Proof. 1. Draw the auxiliary line FG, and let HK be drawn from $H \perp$ to AB. FG

2. At a given point in a straight line a perpendicular to the line can be drawn, and but one. (25)

∴ 3. HK is $\perp$ to FG at H.





Apply the line FG to the line AB, and move it along until HK passes through C. Let D be the point where H falls. Draw the line CD.

1. CD has two points, C and D, which coincide with points in HK by construction.

* From Wells's *Essentials of Plane and Solid Geometry.*

2. But one straight line can be drawn between two points. (Ax. 3.)

∴ 3. CD coincides with HK, and CDB coincides with and is equal to KHG.

1. KHG is a rt. $\angle$.

2. $\angle CDB = \angle KHG$.

∴ 3. CDB is a rt. $\angle$.

1. CDB is a rt. $\angle$.

2. When a line makes a right angle with another line, it is said to be perpendicular to it. (24)

∴ 3. CD is $\perp$ to AB.

Hence one perpendicular can be drawn from the point C to the line AB.

If there can be another $\perp$ from C to AB, let it be CE. Produce CD, making $C'D = CD$. Draw $C'E$.

1. $C'D = CD$ by construction.

ED is $\perp$ to CC' by construction.

2. If lines be drawn to the extremities of a straight line from any point in the perpendicular erected at its middle point, they make equal angles with the perpendicular. (44)

∴ 3. $\angle C'ED = \angle CED$.

1. $\angle C'ED = \angle CED$.

2. $\angle CED$ is a rt. $\angle$ by hypothesis.

∴ 3. $\angle C'ED$ is also a rt. $\angle$.

Add $\angle CED$ and $\angle C'ED$.

1. $\angle CED$ is a rt. $\angle$ by hypothesis.
$\angle C'ED$ is a rt. $\angle$ by proof.

2. If the sum of two adjacent angles is equal to two right angles, their exterior sides lie in the same straight line. (37)

$\therefore$ 3. CEC' is a straight line.

1. CEC' is a straight line by proof.
CDC' is a straight line by construction.

2. But one straight line can be drawn between two points. (Ax. 3.)

$\therefore$ 3. As we *know* that CDC' is a straight line by construction, CEC' cannot be a straight line.

If CEC' is not a straight line, then $CED + C'ED$ is not = to two rt. $\angle$s.

If $CED + C'ED$ is not equal to two rt. $\angle$s, then CED, which is half of this sum, is not a rt. $\angle$.

If CED is not a rt. $\angle$, then CE is not $\perp$ to AB.

As CE is any other possible $\perp$ from C to AB, then CD is the only $\perp$ from C to the line AB.

Hence there can be only one perpendicular from C to the line AB.

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Professor of Mathematics in the Massachusetts Institute of Technology.

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